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An early 20th century handbook on 'meta-analysis': David Brunt's *The Combination of Observations*

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> Hence it follows that the accuracy of the arithmetic mean is $\sqrt{n}$ times the accuracy of a single observation; a result of fundamental importance in the theory of errors. (Brunt,[1] p.10)

The first edition of *The Combination of Observations*[1] by David Brunt (1886–1965) appeared in 1917 and thus nearly 60 years before Eugene Glass's influential paper[2] of 1976. Glass was addressing a problem that Brunt did not consider, namely how does one produce a quantitative synthesis of results from a body of studies, when the type of measurement used varies from study to study. (What has since been referred to as a *Type C meta-analysis*.[3]) Nevertheless, the term *meta-analysis* was introduced by Glass and was even more influential than the technique and is now also used to describe the synthesis of results from studies using a common scale of measurement and, indeed, the majority of what are now called *meta-analyses* are of this sort.

Brunt explains the purpose of his book thus: 'The aim of this book is to give an account of the method of least squares, without entering into elaborate descriptions of instruments or experimental methods' (page v). Least squares already had a venerable history by the time of Brunt's book and is associated with the work of many leading mathematicians of the late 18th and early 19th centuries,[4] in particular Adrien-Marie Legendre (1752–1833) and Karl-Friederich Gauss (1777–1885).

The modern student of least squares will usually approach it as a technique in a subject confusingly referred to as *regression*, in which context it will be developed as a means of finding unique estimates of parameters in an over-determined system of linear equations. The first case that is usually treated is that of fitting a straight line with two unknowns, intercept and slope. However, an even simpler such problem is that of estimating a single constant and this is what meta-analysis does.

By the time of Brunt's book, the theory of least squares was a well-developed subject, as was the particular aspect of it we would now call meta-analysis. For example, George Bidell Airy (1801–1892) had published a book[5] with a title that included the phrase, *The Combination of Observations*, that Brunt was to use for his. Curiously, however, Airy's book is not cited by Brunt, despite the fact that it was still well-known at the time of Brunt's, as is attested to by reference to it in correspondence between Student (1876–1937) and Egon Pearson (1895–1980).[6] (See also Lehmann's discussion.[7]) Furthermore, FJM Stratton (1881–1960), whose help Brunt acknowledges and who was at Caius at the same time as Fisher, taught a course on Combination of Observations,[8] which, given its title, quite possibly made reference to Airy's book. However, Brunt's book was more detailed in its treatment, broader in its scope and generally more ambitious than Airy's, and Brunt may have felt that there was no debt to acknowledge.

David Brunt

> . . . began the writing of his first book, *The combination of observations*, which was published in 1917 and once had a considerable vogue as a practical manual in statistics. (Sutton,[9] p.42)

The quotation above is from Sutton's biography for The Royal Society,[9] upon which this section is based.

David Brunt was born into a Welsh-speaking family in the village of Staylittle, Montgomeryshire in 1886. He was schooled in Welsh until the age of 10, when the family moved to the English-speaking area of Llanhilleth. From that time on, his education was in English, a language to which he rapidly adapted. In 1899, he entered the Abertillery Intermediate School, where he proved to be a star pupil, winning a scholarship to the University College of Wales, Aberystwyth, where he studied physics and mathematics, graduating with a distinguished first class honours in 1907. A scholarship then brought him to Trinity College, Cambridge, where he gained firsts in parts I and II of the mathematical tripos. He initially hoped to be an astronomer, but in 1916 he enlisted with the Royal Engineers (Meteorological Section), and meteorology was the field in which he subsequently made his reputation.

In meteorology, he made contribution to many topics, including atmospheric turbulence, periodicities in European weather, the dynamics of cyclones and anti-cyclones, eddy heat-transfer and predicting night-time minimum temperatures, to name but a few. In 1934, he was appointed Professor at Imperial College London, where he was the head of the only department of meteorology at a British University. He was appointed Fellow of the Royal Society in 1939 and knighted in 1949.

He was an enthusiastic glider pilot and was chairman of the Council of the British Gliding Association from 1935 to 1946. He married Claudia Mary Elizabeth Roberts in 1915. They had one son. Both his wife and son predeceased him. He died in 1965. The Brunt Iceshelf in Antarctica is named after him.

The Combination of Observations

The material of Brunt's book is arranged in 12 chapters. Chapter I, Errors of Observation, makes some general remarks and Chapter II, The Law of Error, provides various derivations of and proofs associated with what would now be called the Normal distribution. Chapter III, The Case of One Unknown, is concerned with the estimation of a population mean when observations have equal weight. Chapter IV, Observations of Different Weight, covers the more general case of unequal precision. Chapters III and IV thus enter into the matter of what we now call *meta-analysis*. The remaining eight chapters address matters that go beyond this and include estimation for more complex models, including correlation, what would now be called multiple regression and Fourier analysis. Chapter VIII, The Rejection of Observations and Chapter IX, Alternatives to the Normal Law of Errors cover matters that are still of interest to statisticians today, although the treatment will seem a little strange.

However, the reader who is interested in medical examples will be disappointed by the book. Of 22 datasets, 12 deal with astronomy, 3 with meteorology, 5 with other physical sciences and 2 with agriculture. Nevertheless, many of the techniques are the same as or similar to those used today on clinical trials. In the remainder of this article, Brunt's approach to meta-analysis will be considered with the help of an example from astronomy given in Chapter IV.

Brunt's recipe for the combination of observations

Figure 1 is taken from the first edition of Brunt's book (but is unchanged in the second edition) as an illustration of Brunt's method of weighted combination of observations. The method of calculation is clearly described by Brunt but some matters are worth clarifying further.

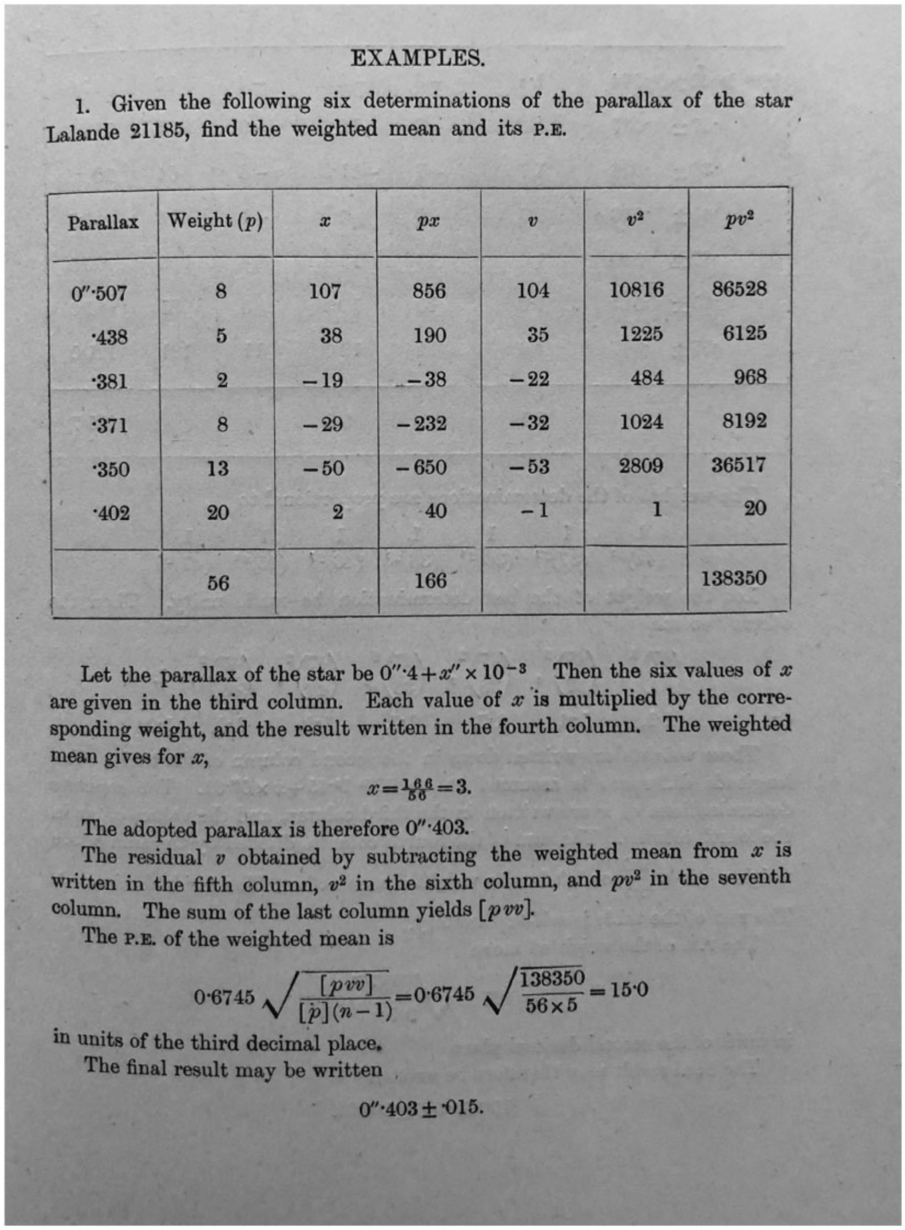
First, by *P.E.*, Brunt means *probable error* (*PE*). This was, at that time, the usual way of expressing precision of observations but was eventually replaced by *standard error* (*SE*). The probability that a random observation from a Normal distribution with mean μ lies within the range $\mu - PE$, $\mu + PE$ is $1/2$. The *PE* is thus simply a multiple of the *SE* and the value of that multiple to four decimal places is 0.6745, and this is the value Brunt uses in his calculations. Note that this value is really only appropriate asymptotically and would be rather poor, given that there are only six observations. This point will be picked up again in the discussion.

Second, the parallax measures being combined are circular but are treated as if they were linear. This is justifiable, given the very small range of values given. Brunt's book includes various types of fitting, including that appropriate to periodic data but simple linear combinations are clearly adequate here.

Third, Brunt does not give any explanation as to where the weights he specifies come from. In modern meta-analysis, these would be inversely proportional to the squares of the standard errors for the estimates being combined. In earlier sections of the book, he has warned the reader that, 'The greatest disadvantage of this method of weighting lies in its arbitrary and personal nature' (p. 64).

Fourth, for ease of calculation, Brunt uses an arbitrary origin of '0.4 seconds' and a scale factor of 1000. This method of calculation was commonly taught up to the 1970s when the author of this article was a student but subsequent generations will probably not have encountered it, as it is no longer needed, due to the availability of personal computers.

Fifth, Brunt estimates the PE using the observed differences between observations. In an earlier part of the book, he discusses what we would now refer to as the fixed effects approach by which the PEs assigned to individual observations could be



EXAMPLES.

1. Given the following six determinations of the parallax of the star Lalande 21185, find the weighted mean and its P.E.

Parallax	Weight (p)	x	px	v	v^2	pv^2
0″·507	8	107	856	104	10816	86528
·438	5	38	190	35	1225	6125
·381	2	−19	−38	−22	484	968
·371	8	−29	−232	−32	1024	8192
·350	13	−50	−650	−53	2809	36517
·402	20	2	40	−1	1	20
	56		166			138350

Let the parallax of the star be $0''{\cdot}4 + x'' \times 10^{-3}$ Then the six values of x are given in the third column. Each value of x is multiplied by the corresponding weight, and the result written in the fourth column. The weighted mean gives for x,

$$x = \tfrac{166}{56} = 3.$$

The adopted parallax is therefore 0″·403.

The residual v obtained by subtracting the weighted mean from x is written in the fifth column, v^2 in the sixth column, and pv^2 in the seventh column. The sum of the last column yields $[pvv]$.

The P.E. of the weighted mean is

$$0{\cdot}6745\sqrt{\frac{[pvv]}{[p](n-1)}} = 0{\cdot}6745\sqrt{\frac{138350}{56\times 5}} = 15{\cdot}0$$

in units of the third decimal place.

The final result may be written

$$0''{\cdot}403 \pm {\cdot}015.$$

Figure 1. Example 1 from page 69 of *The Combination of Observations.*

combined to estimate the PE of their weighted combination. He warns that,

> If the differences between the individual observations that have to be combined are attributable to systematic errors entering into different determinations in different ways, it is clear that the P.E. of a determination can give no clear estimate of the reliability of that determination. (p.68)

He then expresses the view that, ‘it is safer to . . . calculate the P.E. from the residuals’ (p.69) unless there are few determinations, say 3 or 4 to be combined and there is no reason to suspect systematic variation. He does not, however, consider the possibility of adjusting the weights accordingly. The net result is that Brunt proposes what we should now call fixed effect weighting but with a random effects approach to estimating the uncertainty of the estimate.

Discussion

The first (1917) edition of Brunt's book is a scholarly and clear account of a theory of combining observations using least squares that had been developed over more than a century. The glaring omission is Student's introduction of the t-distribution.[10] Brunt quotes a multiplier to calculate the PE of 0.6745. The t-distribution would have to have in excess of 4000 degrees of freedom to give this value to four decimal places. In the example illustrated, he has six observations only and therefore five degrees of freedom. The appropriate multiplier from the t-distribution is then 0.7276. In other words, Brunt's value is accurate to one decimal place, not the four quoted.

He can, however, be forgiven for this oversight. The development of adjustment for uncertainty in the measurement of uncertainty took place over a long period. We now know that Student was anticipated in 1876 by Jakob Luroth.[11] Furthermore, Student's paper received little attention when first published. It was only when his approach was adopted, modified and further developed by RA Fisher in 1925 in *Statistical Methods for Research Workers*[12] that the technique entered mainstream statistics. Its omission in Brunt's first edition is therefore not surprising. However, it is also omitted in the 1931 edition and this means that whereas the book was an excellent account of the art of combining observations when it first appeared, it was seriously out of date by its second edition.

Meta-analysis has a much longer history than one might suppose. In his popular account of the subject,[13] Morton Hunt attributes its origins to Karl Pearson's paper of 1904 on the efficacy of inoculation against typhoid fever.[14] However, it was nearly a century old by then, Adrien Legendre having proposed least squares[15] in 1806 as a means of determining the orbits of comets. Its continued development was then in the subject of astronomy and related physical sciences. Brunt's book is an excellent summary of what had been achieved by the start of the 20th century in that tradition.

Meta-analysis was then further developed in agricultural science (see, for example, work by Yates and Cochran[16]) and then subsequently in education and medicine. At each stage, it seems, something was lost, both by those in the new field of application failing to appreciate what had already been done, and those in the old field being unaware of further developments elsewhere, as Brunt's book illustrates. Nevertheless, even though soon outdated, the book remains an important historical account and can be read with profit, even today.

Declarations

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